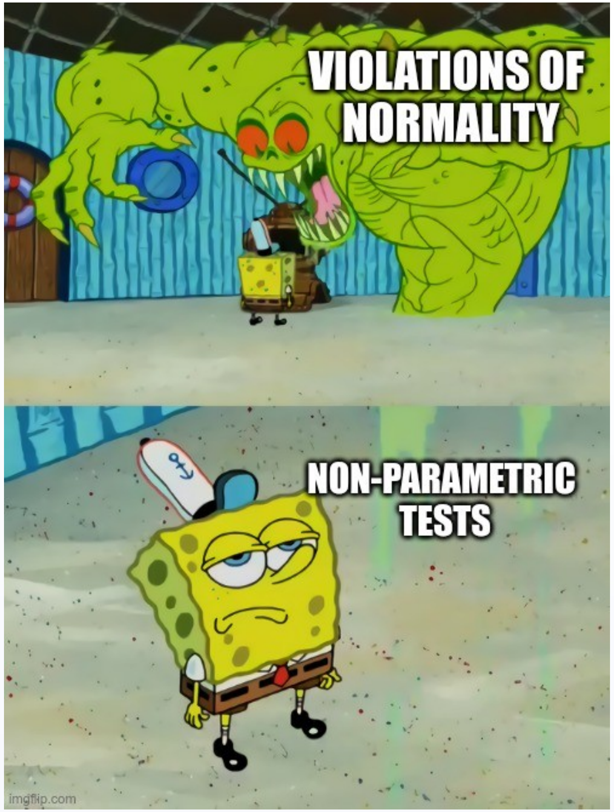
EGM101 – Skills Toolbox

Week 8, Part 5: Non-parametric Tests

When do I use non-parametric tests?

- For parametric tests, we assumed:
  - (Approximately) normal population
  - Continuous data
- This won't always be the case
  - Ordinal data
  - Skewed data
  - Small sample sizes
- When data don't fit assumptions, use non-parametric tests
- With non-parametric tests, usually use median (η) instead of mean (μ)



Wilcoxon signed-rank tests

- Use when we have data that are:
  - Continuous
  - Symmetric
- Test statistic: W^+
  - Alternative versions: W^-, $\min(W^+, W^-)$
- If $n(n+1)/2 > 20$, W-statistics are approximately normally-distributed
  - Distribution with μ_w, σ_w
  - Use z-score, standard normal distribution to find critical values.
- If n is small, use lookup tables to find critical values
- In case of ties, variance becomes more complicated
  - Subtract $(t^3 - t)/48$ from variance for each group of t tied observations

$$\mu_w = \frac{n(n+1)}{4} \qquad \sigma_w^2 = \frac{n(n+1)(2n+1)}{24}$$

$$z = \frac{W^+ - \mu_w}{\sigma_w}$$

Signed ranks

- Ranking differences:
  - One sample: difference to mean/median
  - Two sample: difference between matched pairs
- Steps:
  - Take differences
  - Rank absolute values
  - In case of ties: assign average rank
  - Sum positive (W^+) ranks
- What about zeros?
  - Classical answer: ignore them
  - Alternatively: include them in assigning ranks, exclude them from test statistic

x	*y*	*x - y*	|*x - y*|	*rank*	*rank w/ ties*	*sign*
8	9	-1	1	1	1.5	–
6	7	-1	1	2	1.5	–
2	0	2	2	3	3	+
4	7	-3	3	4	4	–
7	1	6	6	5	6	+
2	8	-6	6	6	6	–
1	7	-6	6	7	6	–
0	7	-7	7	8	8.5	–
8	1	7	7	9	8.5	+
9	1	8	8	10	10	+

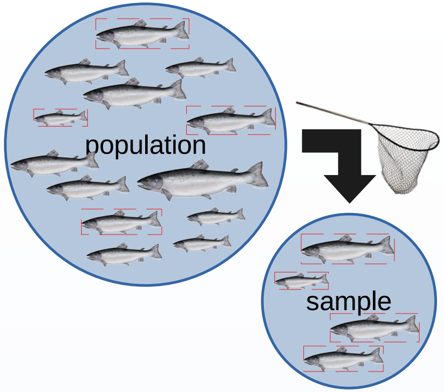
$W^+ = 27.5$

$W^- = 27.5$

One-sample Wilcoxon

- Example: seven salmon
  - Is population median equal to 150 cm?
  - Small sample size → need non-parametric test
- Hypotheses:
  - H_0: $\eta = 150$ cm
  - H_a: $\eta < 150$ cm
- Steps:
  - Subtract η_0 from each observation
  - Rank absolute differences
  - Calculate W^+ (left-tailed test)
- From lookup table: for one-tailed test with $n = 7,\ \alpha = 0.05,\ W_c = 4$
  - Because $W^+ > W_c$, we fail to reject H_0

population: $\eta_0 = 150\,\mathrm{cm}$



length	128	169	178	134	160	154	120
length – η_0	-22	19	28	-16	10	4	-30
|length – η_0|	22	19	28	16	10	4	30
rank	5	4	6	3	2	1	7

$\boxed{W^+ = 13}$

Two-sample Wilcoxon

- Non-parametric counterpart to paired sample t-test
- Steps:
  - Difference paired samples
  - Rank absolute differences
  - Calculate W^+
- Use normal approximation or use a lookup table

Mann-Whitney U-test

- Non-parametric counterpart to independent samples t-test
- Question: is there a difference in the rank sum between two groups?
- Requirements:
  - Random, independent samples
  - At least one sample with $n > 5$
  - Dependent variable is ordinal or numeric (have to rank)
- Calculate statistic U, then either:
  - Look up critical value in a table
  - If enough observations, U is approximately normal

Mann-Whitney U-test

- Hypotheses:
  - H_0: $\eta_1 = \eta_2$
  - H_a: $\eta_1 \neq \eta_2$
- Steps:
  - Rank the values
  - For each variable, sum the ranks
  - Calculate U_1, U_2
- If $n > 20$, U is approximately normally-distributed
  - Expected value, μ_U
  - Standard error, σ_U
- With $z = -1.461$, $p = 0.144 > \alpha = 0.05 \rightarrow$ do not reject H_0

1	42	11	36	34	23	39
2	8	12	41	9	14	

rank	1	2	3	4	5	6	7	8	9	10	11
value	8	9	11	12	14	23	34	36	39	41	42

$$R_1 = 44 \qquad R_2 = 22$$

$$U_1 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2 = 23 \qquad U_2 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1 = 7$$

$$\mu_U = \frac{n_1 n_2}{2} \qquad \sigma_U = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$$z = \frac{\min(U_1, U_2) - \mu_U}{\sigma_U} = \frac{7 - \frac{6*5}{2}}{\sqrt{\frac{6*5(6+5+1)}{12}}} = -1.461$$

Summary

- Non-parametric tests: when we don't assume anything about the distribution of the population
- Non-parametric counterparts to parametric tests:
  - One-sample t-test ↔ One-sample Wilcoxon
  - Paired sample t-test ↔ Two-sample Wilcoxon
  - Independent sample t-test ↔ Mann-Whitney U-Test
- Remember: focus more on <u>why</u>, rather than the formulas

Additional Resources

- Weiss, Chapters 9.6, 10.4, 10.6
- Parametric vs Non-parametric tests, and when to use them [A. Kline]
- Parametric and Nonparametric Tests [DATAtab]
- Parametric vs. Non Parametric Tests [Prof. Essa]