EGM101 – Skills Toolbox

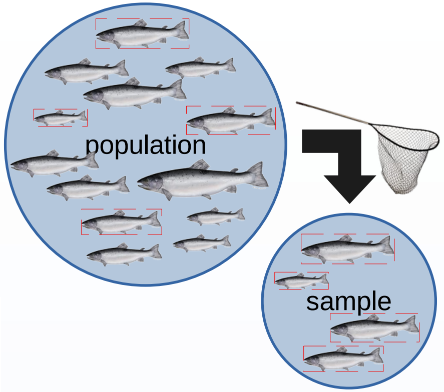
Week 8, Part 3: Parametric tests

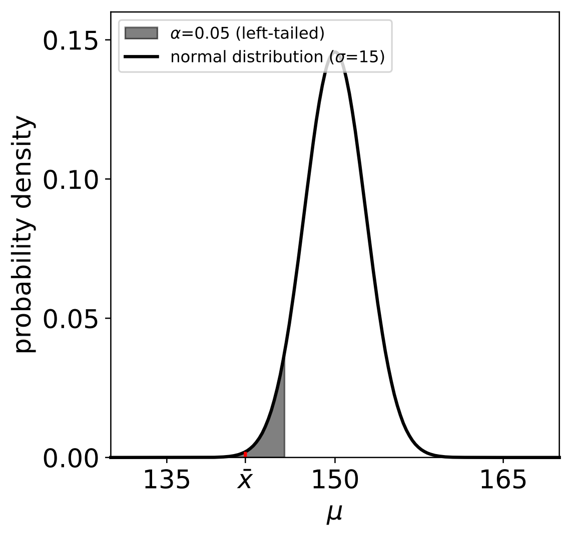
What is a parametric test?

- Parametric test: assume population can be modeled by a probability distribution with fixed *parameters*
  - e.g., normal distribution
  - Example: Z-test
  - Mostly going to focus on Student's t-distribution
- Non-parametric test: don't assume a probability distribution for the population

population: $\mu = 150\,\text{cm}$, $\sigma = 15\,\text{cm}$

sample (n = 30): $\bar{x} = 142\,\text{cm}$, $s = 20\,\text{cm}$

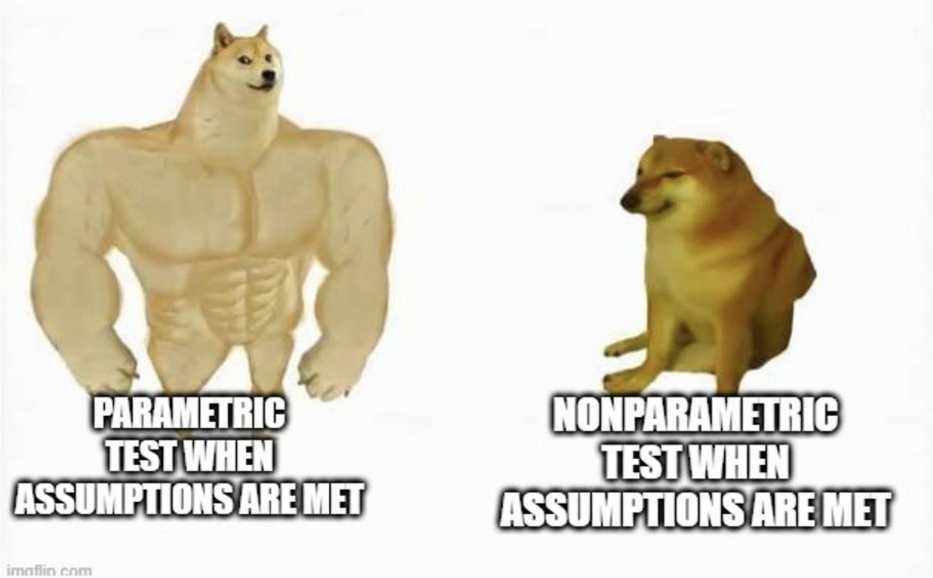




$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{142 - 150}{15/\sqrt{30}} = -2.921$$

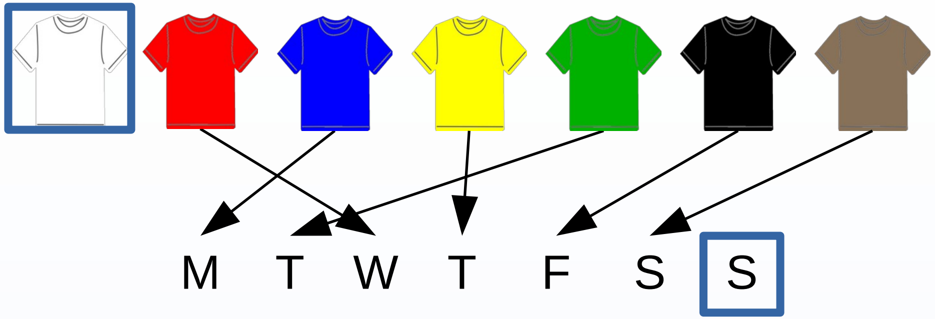
When do I use parametric tests?

- When we are working with normally-distributed, continuous data
  - Remember the Central Limit Theorem!
  - If in doubt, check the data!
- When our observations are independent
- When the variance of the samples is approximately the same
- In general, prefer parametric tests because they tend to have more power



Degrees of Freedom

- Remember: $\bar{x}$, s are *estimates* of μ, σ
- Degrees of freedom (v): the number of *independent* pieces of information that are used to calculate an estimate
  - Example: wearing a different t-shirt every day during the week
  - Only 6 *independent* choices available
- Alternative definition: the number of observations that are free to vary
- Example: calculating sample mean
  - For a given value of $\bar{x}$, only $n-1$ values are "free" to vary
  - In practice, for mean: $v = n - 1$



$\bar{x}=6$

2	1	13	3	7	10	10	7	4	3

$$\sum_{i=1}^{9} x_i = 57 \rightarrow x_{10} = 3$$

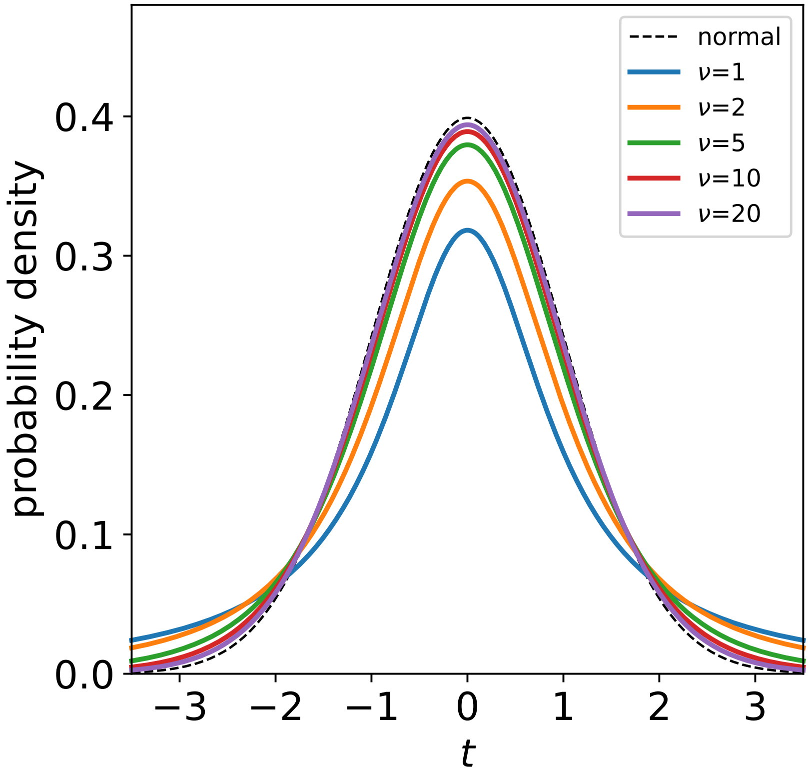
$\bar{x}=10$

2	7	19	18	12	10	16	3	4	9

$$\sum_{i=1}^{9} x_i = 91 \rightarrow x_{10} = 9$$

Student's t-Distribution

- Similar to normal distribution, but takes degrees of freedom (v) into account
  - Shorter peak, larger tails
- As v increases, distribution approaches normal distribution
  - Above $v=20$ or so, essentially the same as the normal distribution
- Use this distribution when:
  - We have a small sample size
  - When we don't know population standard deviation (most of the time)

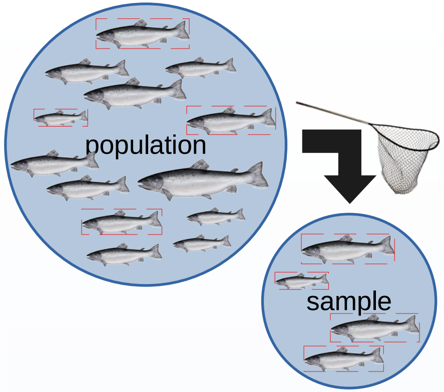


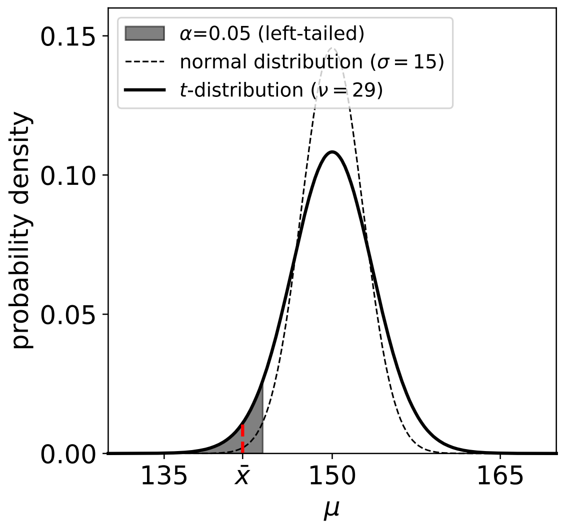
Student's t-Test for a single mean

- Proceed exactly like z-test
  - Instead of normal distribution, use t-distribution with $v = n–1$
- Example: salmon length
- Hypotheses:
  - H_0: $\mu = 150$ cm
  - H_a: $\mu < 150$ cm
- Critical t for $v = 29$, $\alpha = 0.05$: $–1.699$
  - $t < t_c$ ($p < \alpha$) → reject H_0
  - Based on the evidence, μ is most likely less than 150 cm.

population: $\mu = 150\,\mathrm{cm}$

sample ($n = 30$): $\bar{x} = 142\,\mathrm{cm}$, $s = 20\,\mathrm{cm}$





$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

standard error of the mean

$$= \frac{142 - 150}{20/\sqrt{30}}$$

$$= -2.19$$

$$p = 0.018$$

Two samples

- With two samples:
  - Comparison is between the difference of means
  - Divided by estimate of the standard error of the mean, $s_{\bar{x}}$
  - Depends on whether samples are paired or not
- Paired samples:
  - Samples are dependent
  - Each measurement in one sample is paired with a measurement in the other sample
  - Useful for observing changes over time
- Unpaired samples:
  - Samples are independent
  - Each individual in one sample is independent of every individual in the other sample
  - Useful for comparing different populations

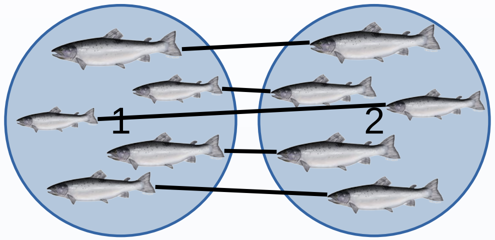
sample difference

hypothesized difference

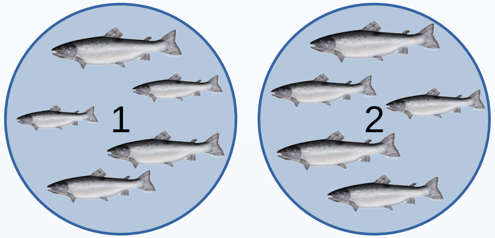
$$t = \frac{\boxed{\bar{x}_1 - \bar{x}_2} - \boxed{(\mu_1 - \mu_2)}}{\boxed{s_{\bar{x}}}}$$

estimate of the standard error of the mean

paired:



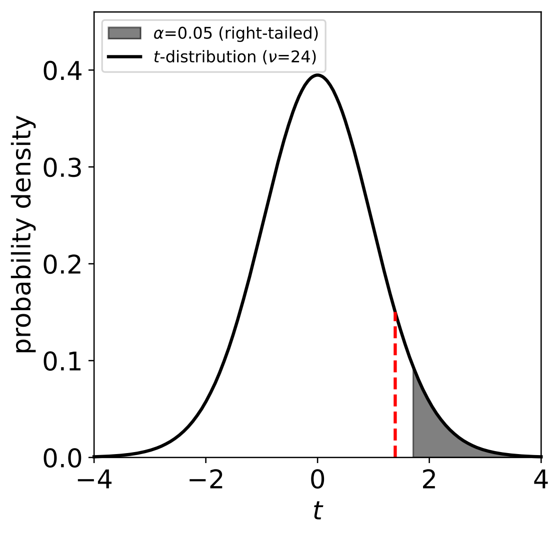
unpaired:



Paired samples

- Example: salmon weighed before, after migration (n=25)
  - Before: 3.5 kg; After: 3.25 kg;
  - s_d = 0.9 kg
- Hypotheses:
  - H_0: $\mu_1 = \mu_2$
  - H_a: $\mu_1 > \mu_2$
- At $\alpha = 0.05$, is there a decrease in the population mean?
  - Right-tailed ($\mu_1 > \mu_2$, or $\mu_1 - \mu_2 > 0$)
  - Degrees of freedom: $v = n - 1 = 24$
  - Critical value, $t_c = 1.711$
- Because $t < t_c$ ($p > \alpha$), we <u>fail to reject</u> H_0

$$t = \frac{\overbrace{\boxed{\bar{x}_1 - \bar{x}_2}}^{\text{sample difference}} - \overbrace{\boxed{(\mu_1 - \mu_2)}}^{\text{hypothesized difference}}}{\underbrace{\boxed{s_d}}_{\text{standard deviation of differences}}/\sqrt{n}}$$



$$\bar{x}_1 = 3.5 \qquad \bar{x}_2 = 3.25$$

$$s_d = 0.9$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_d/\sqrt{n}}$$

$$= \frac{3.5 - 3.25}{0.9/\sqrt{25}} = 1.389$$

$$p = 0.089$$

Unpaired samples

- For unpaired samples, form of $s_{\bar{x}}$ depends on population variances
- If variances are "similar":
  - pooled variance, s_p^2
  - $\nu = n_1 + n_2 - 2$
- If variances aren't "similar":
  - "Welch's t-test"
  - ν: it's complicated…
- Note: don't memorize the formulas, but understand when to use which form

$\frac{1}{2} < \frac{s_1}{s_2} < 2:$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

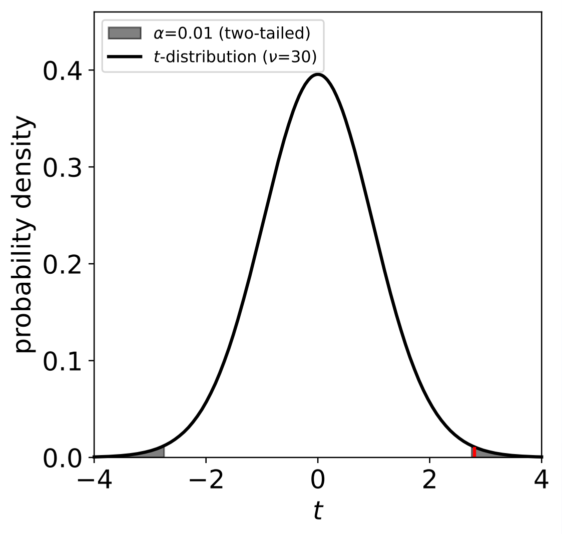
$s_1 > 2s_2$ or $s_2 > 2s_1$:

$$t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{s_{\bar{\Delta}}} \qquad s_{\bar{\Delta}} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Independent samples

- Example: effect of wastewater discharge on fish health
  - Upstream (μ_1): sample of 18 fish
  - Downstream (μ_2): sample of 14 fish
- Hypotheses:
  - H_0: $\mu_1 = \mu_2$
  - H_a: $\mu_1 \neq \mu_2$
- At $\alpha = 0.01$, is there a difference in length between the two populations?
  - Two-tailed (any difference)
  - Degrees of freedom: $\nu = n_1 + n_2 - 2$
- Because $t > t_c$ ($p < \alpha$), we <u>reject</u> H_0

$$t = \frac{\overbrace{\boxed{\bar{x}_1 - \bar{x}_2}}^{\text{sample difference}} - \overbrace{\boxed{(\mu_1 - \mu_2)}}^{\text{hypothesized difference}}}{\underbrace{\boxed{s_p}}_{\text{pooled standard deviation}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \qquad s_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$$



	upstream	downstream
$\bar{x}$ (cm)	23.97	20.67
s (cm)	3.10	3.56
n	18	14

$$t = 2.800 \qquad p = 0.004$$

$$t_c = 2.749$$

Summary

- Parametric tests: when we assume something about the distribution of the population
- Most of the time, we end up using Student's *t*-distribution
- Calculation varies depending on:
  - How many samples (one or two)
  - Independence of samples
  - Population variances
- Remember: focus more on *why*, rather than the formulas

Additional Resources

- Illowsky and Dean, Chapters 9, 10
- Caswell, Chapter 15
- Weiss, Chapters 9, 10
- What are degrees of freedom in statistics? [Minitab Blog]
- Nonparametric tests vs Parametric tests [Jim Frost]
- One-tailed and two-tailed tests [Khan Academy]
- Z-statistics vs T-statistics [Khan Academy]