Slide 1 – Title Slide

Hello and welcome to Week 7, Part 2 of EGM101: More Probability. In this lesson, we'll continue our introduction into the fundamental concepts of probability.

Slide 2 – Complements

In the previous lesson, we introduce some of the basic definitions of probability, like "outcome," "sample space," and "event." In the first part of this lesson, we're going to take a deeper look at how these different concepts relate to each other, and how we can combine events to think about more complicated probabilities.

Hopefully, it won't be too much of a surprise for me to say that a sample space, which we will denote using an uppercase S, is the set of all possible outcomes of an experiment.

Using the example of the number of dots showing on a six-sided die or D6, we can write S like this, using curly brackets to indicate that it's a "set".

And you should also remember that an "event," which we will denote using an uppercase A, is just a combination of different outcomes. So, for example, the event where we roll a D6 and get either a 1 or a 2 can be written like this, where we are again using curly brackets to indicate that we're dealing with a set.

Instead of using this specific example, though, we can also think of this more abstractly, using shapes. So, we could draw S as a rectangle (or any other shape, but my art skills are very simplistic), and A as a gray circle inside of the rectangle. This can also help us think about probabilities, since the probability of A is the proportion of the area of S represented by A – or, the area of A divided by the area of S.

This kind of diagram is known as a Venn Diagram – you are probably more familiar with the example where there are overlapping circles, and don't worry – we'll get to that.

So, if A is this gray area, and S is the whole rectangle, what is the rest of the rectangle? This is something that we call the complement of A, also called "A prime" – it's the set of all possible outcomes in S that aren't part of A.

Using our D6 example, the complement of rolling a 1 or a 2 is rolling a 3 or higher – we can write this using set notation like this, where A prime, or the complement of A, is the set of rolling a 3, 4, 5, or a 6.

Using the Venn Diagram, A prime looks like this – it's all of the area of S that isn't covered by A.

And, because we know that the probability of getting an outcome that is within S is equal to 1 (because S is the set of all possible outcomes, remember), this means that the probability of A, plus the probability of the complement of A, has to equal 1.

Slide 3 – Unions

Sticking with our D6 examples, let’s say now that we have two events – the first, event A, is the same as before: we roll either a 1 or a 2.

The second event, event B, is where we roll an even number – either a 2, a 4, or a 6.

The union of A and B, written like this with a symbol that looks like a small u, is the combination of the outcomes of both event A and event B. Because this is often used to consider the probability of either of these two events happening, you may also see or hear it referred to as “A or B”.

Using set notation, we would write it like this: first, we look at the different outcomes that make up event A and event B, and then we put them together into a single set, like this.

Note that we don’t double-count the outcomes that are part of both events. When we draw the Venn Diagram, the union is the total gray area, not necessarily the sum of the area of the two circles.

But, it’s important to note that events A and B do not have to overlap for us to work with the area – for example, if event A is that the roll is an odd number, and event B is that the roll is an even number, we can still use the union of these two events.

Slide 4 – Intersections

The intersection of two events is all of the outcomes that they share, written using this upside-down u symbol. Because it’s the outcomes in both A and B, you will also see this written as “A and B”.

If A and B have no outcomes in common, the intersection is the “empty” set, denoted as a zero with a line through it, or a pair of curly brackets with nothing in between them.

Using our D6 example, we see that the intersection of A and B – or the possible outcomes that are both less than three and even, has exactly one outcome – a two.

When we look at the Venn Diagram for the intersection, we can see that this is the classic example – it’s the part of A and B that overlaps, shown here in purple.

One final note about the intersection – the intersection of any set with its complement is, by definition, empty – the complement of event A is defined as all of the outcomes that aren’t in A, so they cannot have any outcomes in common. To illustrate this, let’s say that we have another event, event C, that is all outcomes where the roll of our D6 is odd. Since event B is all of the even outcomes, C is also the complement of B.

And, if we write this out explicitly, we see that there are no outcomes in common between events B and C – their intersection is the empty set.

Slide 6 – Mutual Exclusion

If events cannot occur at the same time, they are mutually exclusive – they share no outcomes in common. Another way of saying this is that they are "disjoint" events.

If event A is where we roll a 1 or a 5, and event B is where we roll an even number, we can see that there are no outcomes shared between them – this is an example of two mutually exclusive events.

Because we know that for mutually exclusive events, the intersection of the two events is empty, this also means that the probability of an outcome being in the intersection of the two events has to be zero – because no such outcomes exist, it's not possible for them to happen.

When we draw the Venn Diagram of mutually exclusive events, it looks like this: the two events don't overlap, since they have nothing in common.

We will discuss this more later in the lesson, but it turns out that events being mutually exclusive can be quite useful when we try to calculate compound probabilities. In general, though, unless we know or can show that events are mutually exclusive, we should assume that they are not.

Slide 5 – Conditional Probability

Now, you might be asking, "this is all well and good, but what does it have to do with probability?" And I promise you, it will be clear just as soon as we start to cover more complicated probabilities – right now.

What if we know that a certain event has happened, and we're interested in the chances of some other event happening as well? That's known as a conditional probability, written like this – we read this as "the probability of A, given that B has happened." The event on the right side of the vertical line is the event that has happened, and the event on the left side is the one we are trying to estimate the probability of.

For example, if we roll an even number on a D6 (event B), what is the probability that the value is also less than three (event A)?

You might realize that this is partly the same thing as asking "how many even values are there that are less than three?" - in other words, it will depend on the probability of an outcome being in both events, or the probability of the intersection of the two events. But, we also need to know the likelihood that event B has happened, or the probability of B.

Formally, we can write this like this, where the probability of event A given event B is equal to the probability of the intersection of events A and B, divided by the probability of event B.

So, remembering that the intersection of our example events is just "2", we write the probability of rolling a 2 in the numerator – this is 1 over 6. The probability of rolling an even number is 3 over 6, or 1 over 2 – half of the numbers on the D6 are even, and half are odd.

So, plugging these values in, we see that the probability of event A, given event B, is 1 over 6 divided by 3 over 6, which simplifies to just 1 over 3.

This hopefully illustrates how a conditional probability “reduces” the sample space – it restricts the number of possible outcomes, because we know that event B has to have happened.

Slide 7 – Summary

In this lesson, we’ve seen how one of the ways we calculate or work with probabilities is using sets – the set of all possible outcomes is the sample space, and an event is a set of individual outcomes.

To calculate probability, we can use set notation to help keep track of different outcomes, or we can use something like Venn diagrams, where we visualize events using shapes.

We also started to discuss what happens when events have overlapping outcomes, and introduced the concepts of unions and intersections, along with complements and mutual exclusion.

Finally, we saw how conditional probability tells us about the likelihood of one event happening, given that another event has already happened.

Slide 8 – Additional resources

You can read more about the topics we’ve discussed here in the textbooks – Illowsky and Dean, Chapters 3.2 to 3.4; Caswell, Chapter 8; or Weiss, Chapters 5.1 to 5.3.

That’s all for this lesson – I hope you found it interesting, and if you have any questions, please don’t hesitate to e-mail me or post in the discussion forum on blackboard. Bye!