EGM101 – Skills Toolbox

Week 7, Part 3: Even More Probability

Independent Events

- Independent: knowing that one event happens does not affect the probability of the other
  - If it does, the events are dependent
- Need to show <u>only one</u> of the following:
  - $P(A|B) = P(A)$
  - $P(B|A) = P(B)$
  - $P(A \cap B) = P(A) \cdot P(B)$

Sampling with/without replacement

- Example: drawing cards from a standard 52-card deck
- With replacement: each time we draw a card, we put it back
  - After each event, the probability remains the same
  - Outcomes are independent
- Without replacement: each time we draw a card, we keep it
  - Probability changes after each event
  - Outcomes are not independent

♠: 13 ♥: 13 ♣: 13 ♦: 13

$$P(♥)=\frac{13}{52}=0.25$$

$$P(♥)=\frac{13}{52}=0.25$$

$$P(♥)=\frac{13}{51}=0.2549$$

$$P(♥)=\frac{12}{50}=0.24$$

The Multiplication Rule

- If A and B are events on a sample space:

$$P(A \cap B) = P(B) \cdot P(A|B)$$

  - Note that this is our conditional probability from earlier

- Remember: if A, B are independent:

$$P(A|B) = P(A)$$

- And so:

$$P(A \cap B) = P(A) \cdot P(B)$$

The Addition Rule

- If A and B are events on a sample space:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Remember that if A and B are mutually exclusive:

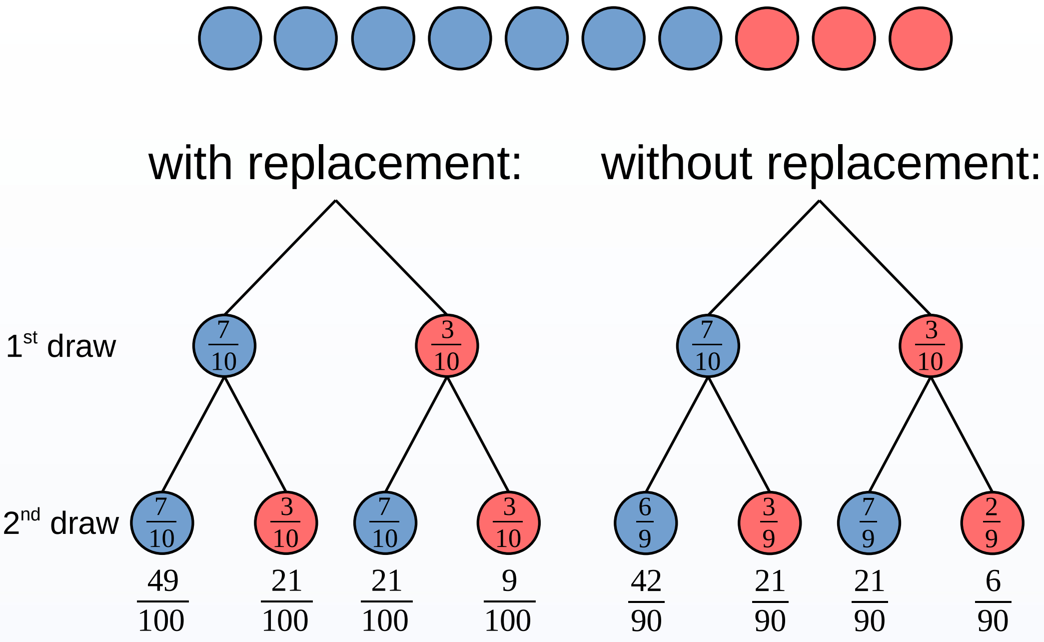
$$P(A \cap B) = 0$$

- And then:

$$P(A \cup B) = P(A) + P(B)$$

Tree Diagrams

- Help visualize/determine the outcome of an experiment
  - Branches labeled with frequencies/probabilities
  - Using the multiplication rule, calculate the probability of each branch at bottom of tree
- Assuming order doesn't matter, what is the probability that after 2 draws, we have:
  - 2 blue marbles?
  - 1 blue, 1 red?
  - 2 red?



With replacement:

$$P(BB)=\frac{49}{100}$$

$$P(BR)=\frac{42}{100}$$

$$P(RR)=\frac{9}{100}$$

Without replacement:

$$P(BB)=\frac{42}{90}$$

$$P(BR)=\frac{42}{90}$$

$$P(RR)=\frac{6}{90}$$

Contingency Tables

- Displays sample values in relation to two different variables
  - May be dependent on each other
- Helps determine conditional probabilities
- Example: fish preferences by pet type
  - What is P(dog)?
  - What is P(prefers salmon)?
  - What is P(prefers salmon | dog)?
  - What is P(dog | prefers salmon)?

	Salmon	Tuna	Total
Dog	39	10	49
Cat	21	30	51
Total	60	40	100

$$P(\text{dog}) = \frac{\text{number of dogs}}{\text{number of animals}} = \frac{49}{100} = 0.49$$

$$P(\text{prefers salmon}) = \frac{\text{number prefer salmon}}{\text{number of animals}} = \frac{60}{100} = 0.60$$

$$P(\text{prefers salmon}|\text{dog}) = \frac{\text{number of dogs that prefer salmon}}{\text{number of dogs}} = \frac{39}{49} \approx 0.796$$

$$P(\text{dog}|\text{prefers salmon}) = \frac{\text{number of dogs that prefer salmon}}{\text{number of prefers salmon}} = \frac{39}{60} = 0.65$$

Summary

- Key concept: whether or not events are independent
  - Changes how we analyze probability
  - Have to think about how we sample (observe)
- Useful tools for understanding, calculating probabilities:
  - Tree diagrams: dependent events
  - Contingency tables: conditional probability

Additional Resources

- Illowsky and Dean, Chapters 3.2 – 3.4
- Caswell, Chapter 8
- Weiss, Chapters 5.1–5.3
- Independent and dependent events [Khan Academy]