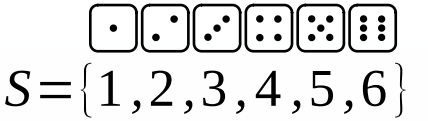
EGM101 – Skills Toolbox

Week 7, Part 2: More Probability

Complements

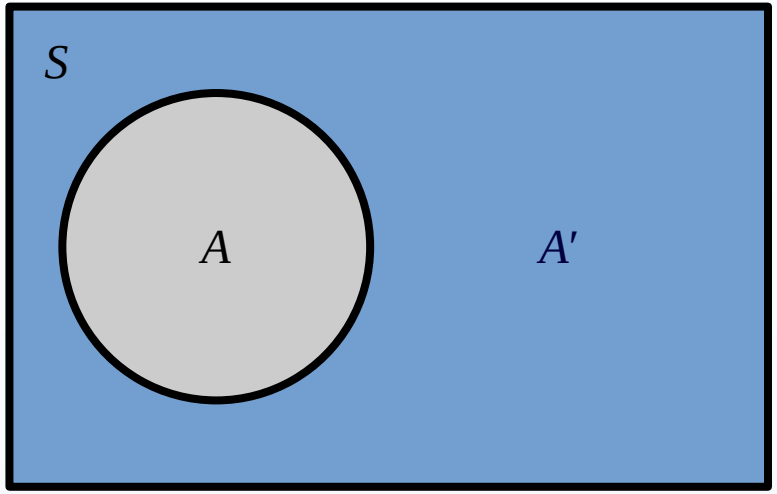
- Recall:
  - Sample space, S: all possible outcomes of experiment
  - Event, A: a combination of outcomes within S
- Complement (A'): all possible outcomes in S that are not in A
- Remember that $P(S) = 1$
- So:

$$P(A)+P(A')=1$$



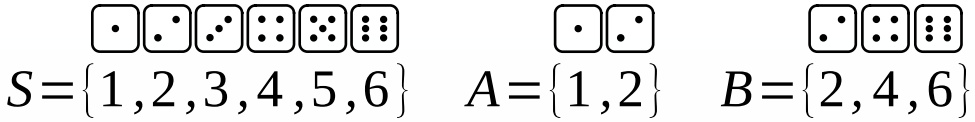
$$S=\{1,2,3,4,5,6\} \quad A=\{1,2\}$$

$$A'=\{3,4,5,6\}$$

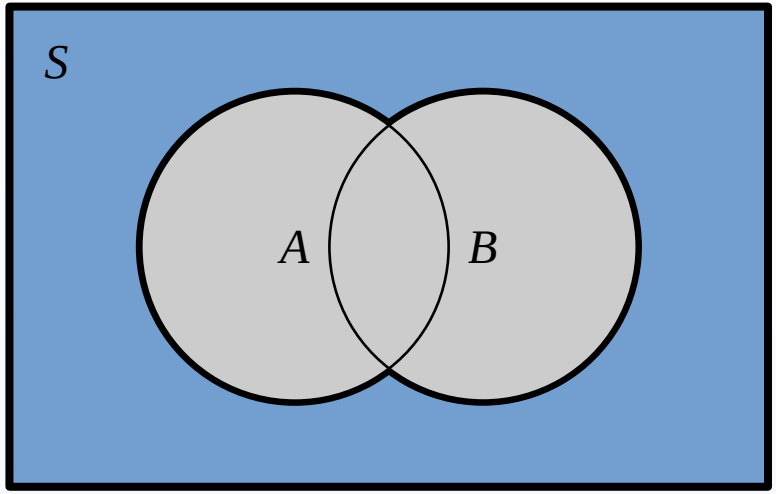


Unions

- Union ($A \cup B$): combination of the outcomes of A or B
  - Can also write this as A or B
- Note: we don't double-count outcomes that are part of both events
- A and B do not necessarily overlap



$$A \cup B = \{1,2\} \cup \{2,4,6\} = \{1,2,4,6\}$$



Intersections

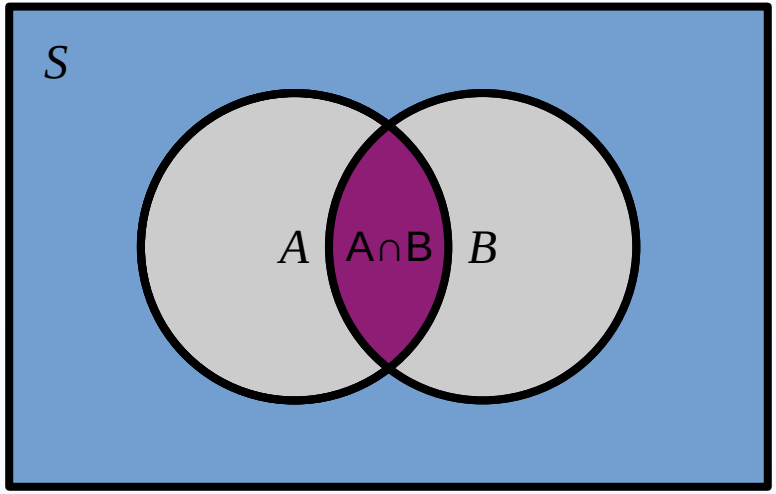
- Intersection ($A \cap B$): all of the outcomes in both A and B
  - Can also write this as A and B
  - If no shared outcomes: *empty set* ($\varnothing$)
- Note that intersection of A, A' is empty:

$$C = \{1,3,5\} = B'$$

$$B \cap C = \{2,4,6\} \cap \{1,3,5\} = \{\} = \varnothing$$

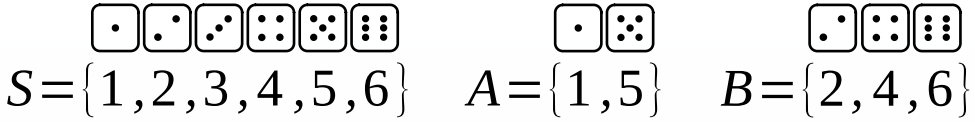
$$S = \{1,2,3,4,5,6\} \quad A = \{1,2\} \quad B = \{2,4,6\}$$

$$A \cap B = \{1,2\} \cap \{2,4,6\} = \{2\}$$

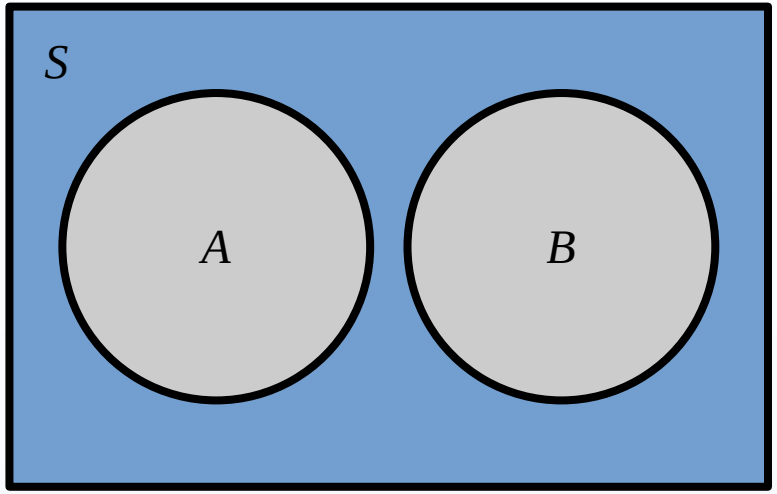


Mutual Exclusion

- If events cannot occur at the same time, they are mutually exclusive
  - They share no outcomes in common
  - Also known as disjoint
- If A, B are mutually exclusive:
  - $A \cap B = \{\}$
  - $P(A \cap B) = 0$
- Unless we know/can show that events are mutually exclusive, we assume that they are not.

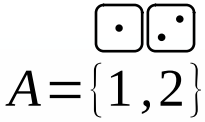


$$A \cap B = \{1,5\} \cap \{2,4,6\} = \{\} = \emptyset$$

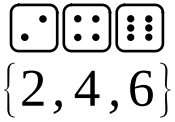


Conditional Probability

- Conditional probability, $P(A|B)$:
  - Given that event B has happened, what are the chances that event A has also happened?
- Example: if we roll an even number (event B), what is the probability that the value is less than 3 (event A)?
  - First: what are the even values in A? ($A \cap B$)
  - Second: what are the chances of rolling an even number? ($P(B)$)
- *Reduces* sample space, because we're restricted to outcomes in event B



$$A = \{1, 2\}$$



$$B = \{2, 4, 6\}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\{2\})}{P(\{2,4,6\})}$$

$P(\{2\}) = \frac{1}{6}$

$P(\{2,4,6\}) = \frac{3}{6}$

$$P(A|B) = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$

Summary

- One of the ways we calculate probabilities is using sets:
  - Set notation
  - Venn diagrams
- Events sometimes have overlapping outcomes
- Conditional probability: what is the probability of one event, given that another event has happened?

Additional Resources

- Illowsky and Dean, Chapters 3.2 – 3.4
- Caswell, Chapter 8
- Weiss, Chapters 5.1–5.3